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## Testing string theory via Born–Infeld electrodynamics?

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## ABSTRACT

We consider the feasibility of an experiment to measure the string parameter  $\alpha'$ . The proposal relies on the stringy prediction that low-energy electrodynamics is described by a Born–Infeld Lagrangian.

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## 1. Introduction

One of the well-known predictions of string theory is that the low-energy effective Lagrangian describing electromagnetism should be of the Born–Infeld (BI) type [1–6]. In particular, if we regard our universe as a D3-brane, open strings attached to the brane may couple to a  $U(1)$  field at the end of the string so that the lowest-order contribution to the partition function – after integrating out the string degrees of freedom allowed by the Dirichlet boundary conditions in the path integral – is given by an action containing the Lagrangian density

$$\mathcal{L} \propto \sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} \quad (1)$$

where  $\mu, \nu = 0, 1, 2, 3$ , and we assume a flat target spacetime. Since the value of the constant in (1) will not enter any of our results, the above will be taken as an equality.

Lagrangians of the BI type have been studied in great detail by a number of researchers. In particular, Boillat [7] and Plebanski [8] have shown that BI is unique among all nonlinear theories

of electromagnetism – with the exception of a singular Lagrangian – in that wave propagation in a background field is free of birefringence: all polarizations propagate with the same speed. This is in contrast to the effective theory obtained from QED, where the speed of propagation depends on the polarization (the difference in speed between the two polarizations is proportional to  $\alpha^2/m^4$ , with  $\alpha$  the fine structure constant and  $m$  the electron mass). Unfortunately, this uniqueness of BI also makes it invisible to experiments such as the optical rotation measurements of the PVLAS Collaboration [9–11].

In recent years several interesting proposals for laboratory experiments sensitive to BI effects have appeared in the literature [12–14]. Even though these ideas can be adapted to our present purposes, we will base our discussion on a novel effect which does not rely on strong fields that remain constant and homogeneous over large scales, or on materials that will most likely experience a breakdown at the high field strengths required by the proposals.

## 2. Wave propagation in BI electrodynamics

As shown by Boillat [7] and Plebanski [8] (see also [15,16]), the propagation of weak disturbances in a nonlinear theory with Lagrangian  $\mathcal{L} = \mathcal{L}(\mathcal{F}, \mathcal{G})$ , where  $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$  and  $\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$ , may be formulated in terms of an effective metric  $g_{\mu\nu}$  such that

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the wave vector  $k^\mu$  is null with respect to that metric,

$$g_{\mu\nu}k^\mu k^\nu = 0 \quad (2)$$

For the Lagrangian (1), the effective metric is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2\pi\alpha'}{\sqrt{1+8\pi^2\alpha'^2\mathcal{F}}} F_{\mu\lambda} F^\lambda{}_\nu \quad (3)$$

and leads to the speed of propagation

$$v = (1 + \mathcal{B}^2)^{-1} [\hat{\mathbf{k}} \cdot (\mathcal{E} \times \mathcal{B}) + \{[\hat{\mathbf{k}} \cdot (\mathcal{E} \times \mathcal{B})]^2 + (1 + \mathcal{B}^2)[1 + (\hat{\mathbf{k}} \cdot \mathcal{B})^2 - (\hat{\mathbf{k}} \times \mathcal{E})^2]\}^{-1/2}] \quad (4)$$

with  $\mathcal{E} = 2\pi\alpha'\mathbf{E}$  and  $\mathcal{B} = 2\pi\alpha'\mathbf{B}$ . As pointed out by Bialynicki-Birula [15], the  $\hat{\mathbf{k}} \cdot (\mathcal{E} \times \mathcal{B})$  term implies that the speed of the wave depends on the direction of propagation. While this effect could in principle be used to design interference experiments to test the prediction (4) (by, say, splitting a beam in two and sending each partial beam in opposite directions with respect to  $\mathbf{E} \times \mathbf{B}$  before recombining them), it has the drawback that it depends on the presence of a strong electric field.

An intriguing alternative is to set up a Michelson interferometer in an external magnetic field so that one arm is perpendicular to the field and the other parallel to the field. Since for  $\mathbf{E} = 0$  the speed (4) reduces to

$$v = \sqrt{1 - \frac{(\hat{\mathbf{k}} \times \mathcal{B})^2}{1 + \mathcal{B}^2}} \quad (5)$$

the beam component propagating in a direction parallel to the field will do so at the speed of light, whereas the component propagating in a direction perpendicular to the field will have  $v < 1$ . The effect may be viewed as due to the presence in the interferometer arm perpendicular to the field of a medium with refractive index  $n \approx 1 + \mathcal{B}^2/2$ . After reflection at the end of the arms, the beams should therefore arrive at the vertex of the interferometer with a difference in optical path length  $\Delta L \approx 4\pi^2 L(\alpha'B)^2$ . Under the most favorable conditions imaginable – a LIGO-type interferometer ( $\Delta L/L \sim 10^{-22}$ ) in orbit around a strong-field ( $B \sim 10^{10}$  T) neutron star, say – this experiment would be sensitive to a string parameter  $\alpha' \sim 10^{-6}$  GeV $^{-2}$  (equivalently, a string length  $\ell_s \sim 10^{-19}$  m or a string scale  $M_s \sim 1$  TeV).

### 3. Deflection of light in BI electrodynamics

We shall now discuss a different type of effect leading to a more viable experimental setup. Consider a magnetic field of the form  $\mathbf{B} = B(r)\hat{\mathbf{z}}$ , with  $r$  the radial distance in plane polar coordinates. From the symmetries of the metric it follows that  $k_0 \equiv \omega$ ,  $k_\phi \equiv \delta$ , and  $k_z$  are conserved quantities. If we prepare a beam so that  $k_z = 0$ , (2) and (3) yield the following equation for the trajectory of the beam

$$\left(\frac{dr}{d\phi}\right)^2 = \left(\frac{\omega}{\delta}\right)^2 \frac{r^4}{1 + \mathcal{B}^2} - r^2 \quad (6)$$

where  $\mathcal{B} \ll 1$  for fields accessible in the laboratory. For a field behaving as a dipole in the plane of propagation,  $B = m/r^3$ , we may approximate the above as

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{a^2} \left(1 - \frac{\mu^2}{r^6}\right) - r^2 \quad (7)$$

with  $a = \delta/\omega$  and  $\mu = 2\pi\alpha'm$ .

The solution to Eq. (7) involves the Jacobi elliptic function  $\text{cn}(\phi, k)$ :

$$r = a \sqrt{\frac{q + p \text{cn } 2\gamma\phi}{1 - \text{cn } 2\gamma\phi}} \quad (8)$$

All the quantities in (8) may be expressed in terms of

$$x = \frac{1}{3}(1 + s + s^{-1}) \quad (9)$$

where

$$s = \left[ \sqrt{1 + \left(3\sqrt{3}\pi \frac{\alpha'm}{a^3}\right)^2} + 3\sqrt{3}\pi \frac{\alpha'm}{a^3} \right]^{2/3} \quad (10)$$

as follows:

$$q = \sqrt{x(3x-2)} + x \quad (11)$$

$$p = \sqrt{x(3x-2)} - x \quad (12)$$

$$\gamma = [x(3x-2)]^{1/4} \quad (13)$$

and the modulus of the elliptic function  $\text{cn}$ ,

$$k^2 = \frac{1}{2} \left[ 1 - \frac{1}{2} \frac{3x-1}{\sqrt{x(3x-2)}} \right] \quad (14)$$

The solution (8) represents a photon coming in from infinity for  $\phi = 0$ , reaching closest approach ( $r = a\sqrt{x}$ ) at  $\gamma\phi = K(k)$ , and leaving toward  $r = \infty$  again for  $\gamma\phi = 2K(k)$ , with  $K(k)$  the complete elliptic integral of the first kind. It follows that the trajectory deviates from a straight line by the amount

$$\Delta\phi = \frac{2K(k)}{\gamma} - \pi \quad (15)$$

Under laboratory conditions  $k \approx 0$ , so

$$\Delta\phi \approx -\frac{15\pi^3}{4} \left( \frac{\alpha'm}{a^3} \right)^2 \quad (16)$$

It is intuitively clear that one should expect this deflection to be small, but it could be substantially magnified by generating the magnetic field with a solenoid, say, and placing the solenoid between plane mirrors so that the  $z$ -axis, the axis of the solenoid, and the planes of the mirrors are mutually parallel. After  $N$  reflections of the beam propagating in the  $x$ - $y$  plane, the above result would be multiplied by this factor, which could result in an enhancement of (16) by four orders of magnitude [11].

At closest approach,  $r = r_0$  and  $dr/d\phi = 0$ , (7) shows that  $r_0 \sim a$  even for the largest fields. Then, inserting the number of reflections  $N$  and denoting by  $B_0$  the field at closest approach,

$$\Delta\phi \approx -\frac{15\pi^3}{4} N \left( \frac{\alpha'B_0 r_0^3}{a^3} \right)^2 \approx -\frac{15\pi^3}{4} N (\alpha'B_0)^2 \quad (17)$$

In terms of the string scale  $M_s$ ,

$$\Delta\phi \approx -\frac{15\pi^3}{4} N \left( \frac{B_0}{M_s^2} \right)^2 \quad (18)$$

For  $B_0 = 10$  T,  $N = 10^4$  as in the PVLAS Collaboration runs, and a LIGO-level sensitivity of  $10^{-22}$ , the effect would be measurable provided  $M_s \lesssim 0.5$  GeV ( $\alpha' \gtrsim 4$  GeV $^{-2}$ ). For single-pass events in which the deflection is due to a  $B_0 = 10^{10}$  T neutron star, a deflection comparable to that due to gravitational microlensing ( $\sim 1$  milliarcsec) is achieved for a similar value of  $M_s$ . The ideal case,  $B_0 = 10^{10}$  T and  $\Delta\phi = 10^{-22}$ , requires  $M_s \lesssim 1.5$  TeV ( $\alpha' \gtrsim 4 \times 10^{-7}$  GeV $^{-2}$ ), well within the purview of low-scale string models with a D3-brane and six Fermi-size compact dimensions [17–19].

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